Elimination of Square Roots from Ballistics Equations

RALPH M. TOMS*
Oregon State University, Corvallis, Ore.

Nomenclature

X = range
Z = altitude
t = time
H = drag function
g = gravitational constant
c = ballistics parameter
ρ = local air density
V = instantaneous speed
C_D = ballistics coefficient
M = Mach number
a = local speed of sound

Introduction

THIS Note contains a simple procedure which eliminates the necessity for computing square roots in the numerical solution of ballistics equations used for endo-atmospheric trajectory prediction. This results in an appreciable reduction in computer time for those machines which do not have a hard-wired square root function. The technique was developed for use in an airborne weapon delivery system where computer time was at a premium.[1,2] The procedure should prove useful in other ballistic applications.

Typical Ballistic Model

To illustrate the principle involved it is sufficient to consider a particular set of ballistics equations. Under several standard simplifying assumptions the equations of motion for an endo-atmospheric ballistic object can be written as

$$\ddot{X} + H\dot{X} = 0$$
$$\ddot{Y} + H\dot{Y} = -g \quad (1)$$

(See Ref. 3 for a derivation). The drag function H is given by

$$H = c\rho V C_D \quad (2)$$

The basic relation for speed is

$$V = (\dot{X}^2 + \dot{Y}^2)^{1/2} \quad (3)$$

The ballistics coefficient C_D is a function of Mach number and is usually given in tabular form.

In the step-by-step numerical integration of the system (1) the value of H is required several times per integration step. The computation of H requires a corresponding value for Mach number so that the appropriate value of C_D can be calculated. Mach number is given by

$$M = V/a \quad (4)$$

This means that V must be computed via a square root subroutine several times per integration step. In this context the square root function requires a relatively large amount of computer time. The need for a square root computation can be eliminated by a simple transformation of variables.

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* Senior Specialist Engineer, Department of Mathematics; on leave from Boeing Aerospace Company on Boeing Doctoral Fellowship Program, 1972–73.

Reformulated Ballistic Model

Suppose that we multiply the denominator and numerator of Eq. (2) by a and use the definition of Mach number in Eq. (4) so that H becomes

$$H = c\rho a M C_D \quad (5)$$

Since C_D is a known function of M it is easy to obtain MC_D as a function of M^2. For instance, if C_D is given in tabular form we can readily construct a new table giving MC_D vs M^2. Once this has been done Eqs. (1) can be numerically integrated without computing any square roots since H can be computed using V^2 rather than V.

Discussion

The computation time saved using the previous transformation is, of course, highly dependent on the particular computer used and the nature of the square root subroutine. In the weapon delivery algorithm discussed in Ref. 2 the computation time was reduced approximately 15% by using the suggested transformation.

The transformation can also be used to remove square root terms from the state-vector used to estimate ballistic re-entry vehicle parameters. In particular, terms involving V can be eliminated from the transition matrix used to update the covariance equation.[4]

References

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Mass Properties of Sphere-Cone Entry Vehicles

WILLIAM J. BOOTLE*
AVCO Systems Division, Wilmington, Mass.

Introduction

IN the initial tradeoffs for an entry vehicle design it is often necessary to know the vehicle mass properties to evaluate dynamic behavior. These are not generally available and some quick method for estimating their value is desirable. Complications arise because a minimum level of static stability is usually required which dictates the mass distribution within the vehicle, while booster payload capability often places

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* Senior Staff Scientist, Aerodynamics/Flight Mechanics Dept., Applied Technology Directorate. Member AIAA.

further restrictions on the design. Thus, there is an involved relationship between axial c.g. location x_{cg}, vehicle weight W, and the moments of inertia I_x and I_y.

The problem can be solved in the case of sphere-cone shapes by postulating that the density at any station x from the nose can be approximated by the relation $\rho = \rho_o e^{Kx} (K < 0)$ i.e., the heaviest concentration of matter occurs at the nose tip where the density ρ_o cannot exceed that of the common ballast material tungsten (1204 lb/ft^3). Maximum efficient utilization of internal volume is assumed with no allowance for voids or the effects of shell thickness. One may then obtain by simple integration a set of nondimensional equations for the parameters of interest as functions of the truncation ratio a/L and KL.

Equations and Design Curves

Center of gravity location

$$X_{cg}/L = [e^{KL}K^2L^2 + BKL(a^2/L^2) + C(2a/L - 3/KL)]/D \tag{1}$$

where

$$A = e^{KL} - 1, \qquad B = e^{KL}(KL - 1) + 1$$
$$C = e^{KL}(K^2L^2 - 2LK + 2) - 2 \tag{2}$$
$$D = AK^2L^2(a^2/L^2) + 2BKL(a/L) + C$$

Weight parameter

$$W/\pi\rho_o L^3 \tan^2\theta_c = D/K^3L^3 \tag{3}$$

Roll moment of inertia parameter

$$2gI_x/\pi\rho_o L^5 \tan^4\theta_c = \{[Aa^4/L^4 + e^{KL}(4a/L + 1)]/KL + 4(Ba^3/L^3 - e^{KL})K^2L^2 + 6C \times [K^2L^2(a^2/L^2) - 2KL(a/L) + 2]/K^5L^5\} \tag{4}$$

Pitch moment of inertia parameter

$$gI_y'/\pi\rho_o L^5 \tan^2\theta_c = \{e^{KL}(2a/L + 1)/KL + 4e^{KL}/K^2L^2 + C[K^2L^2(a^2/L^2) - 6KL(a/L) + 12]/K^5L^5 - [e^{KL}K^3L^3 + BK^2L^2(a^2/L^2) + 2CKL(a/L) - 3C]^2/DK^5L^5\} \tag{5}$$

where I_y' represents part of the total I_y which is given by

$$I_y = I_y' + I_x/2 \tag{6}$$

The preceding equations are not valid at $KL = 0$; that is the special case of a truncated cone with uniform mass for which standard textbook formulae are available. They have been solved on the IBM 360 computer for a representative range of a/L and KL and the results are shown in Figs. 1–4. These

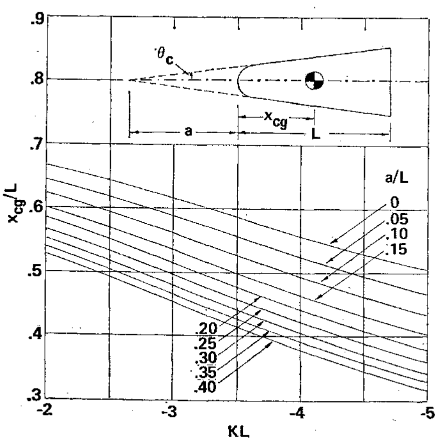

Fig. 1 Nondimensional c.g. location vs KL and a/L.

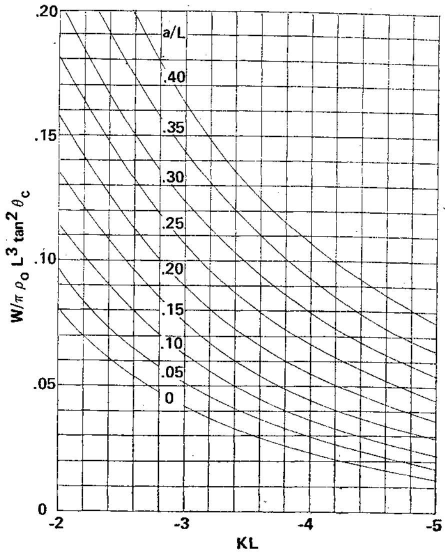

Fig. 2 Nondimensional weight parameter vs KL and a/L.

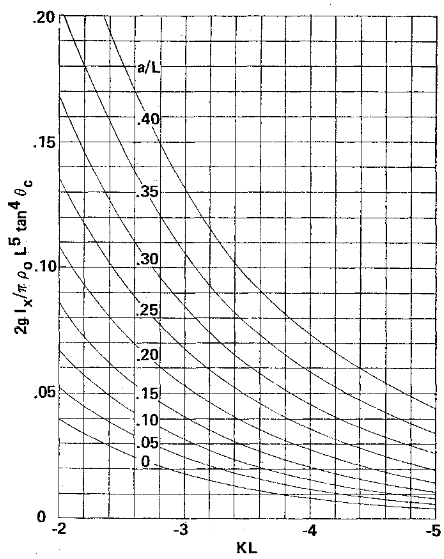

Fig. 3 Nondimensional roll inertia parameter vs KL and a/L.

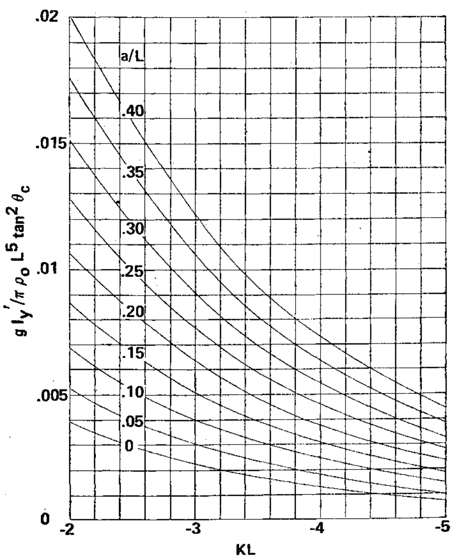

Fig. 4 Nondimensional pitch inertia parameter vs KL and a/L.

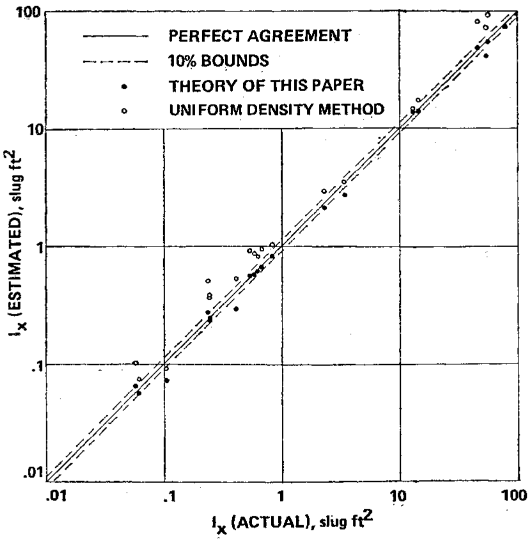

Fig. 5 Comparison between estimated and actual I_x values.

design curves are used as follows. For a given a/L read into Fig. 1 at the required x_{cg}/L and determine KL (it is not necessary to solve for K explicitly). Read into Fig. 2 at this KL, determine the weight parameter, and by appropriate substitution estimate either the maximum value of W using the tungsten value for ρ_o or the maximum allowable ρ_o if W has been defined by other system constraints. Proceed in similar fashion to Figs. 3 and 4 thereby estimate I_x, I_y' and, hence, I_y.

Accuracy of the Method

The equations have been tested against a large matrix of cone-sphere entry vehicles having known mass properties. A KL value and ρ_o were selected commensurate with the known c.g. location and weight. I_x and I_y were then estimated and plotted vs their actual values. The results are shown in Figs. 5 and 6, and good agreement is seen to exist, with 75% of I_x and 65% of I_y within $\pm 10\%$ of the line of perfect agreement. Those I_x cases outside these bounds are on the low side which is correct from the standpoint of a conservative approach to roll dynamics. Shown also are the results obtained by using a uniform density distribution. This is a common technique for estimating I_x and I_y in preliminary design studies wherein I_y is calculated about the c.g. with uniform mass and then transferred to the true c.g. using the parallel axis theorem. Except in a few isolated cases the agreement is poor.

Although the method described here is intended for sphere-cones it can be used for shapes of close resemblance such as blunt biconics. If the vehicle has an aft cover such as a hemisphere, the curves may be used without significant error provided one uses total vehicle length for L rather than the length to maximum diameter. In general the curves are restricted to vehicles with base diameters in excess of 5 in. Below this value the volume constraints imposed by shell thickness and the dominant contribution of the shell to the total weight become significant.

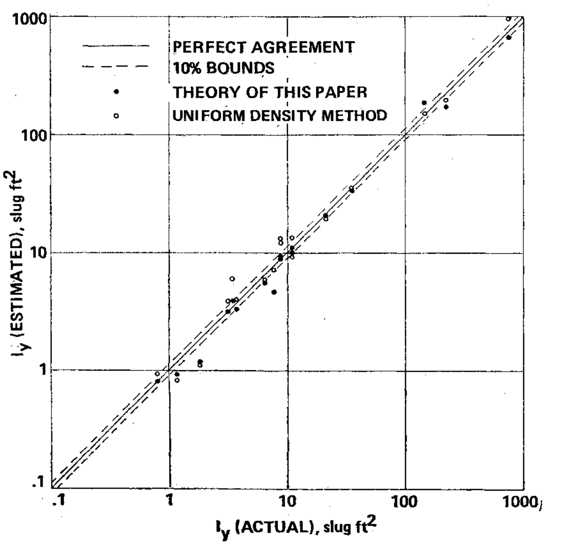

Fig. 6 Comparison between estimated and actual I_y values.

An Advanced Technique for the Prediction of Decelerator System Dynamics

THEODORE A. TALAY,* W. DOUGLAS MORRIS,*
AND CHARLES H. WHITLOCK†
NASA Langley Research Center, Hampton, Va.

Introduction

IN the field of nonstationary aerodynamics, few subjects approach the degree of difficulty of the parachute opening process. The prediction of the influence of a decelerator system on an entry vehicle has heretofore been largely an empirical process. With parachute applications now probing far past the realm of previous test experiences—as evidenced in Project Viking—it has become useful to model the very complex behavior of parachute opening dynamics for extrapolation to new flight conditions. This has been aided by the rapid advancements in computer capabilities to accept complex models. This Note describes an advanced two-body, six-degree-of-freedom, computerized, analytical model for the parachute opening process and presents a sample case.

Background

The interval from the beginning of parachute deployment until a stable inflation represents a transient regime of greatest concern in terms of forces and moments on an entry vehicle. These forces and moments influence the parachute system design, vehicle trajectory, and the guidance and control system.

For Viking, the opening process consists of an unfurling phase, where the parachute deploys in a lines-first manner following mortar fire, and a canopy inflation phase—the inflation phase is assumed to begin with the parachute fully strung out. The canopy opens to full inflation and is usually followed by a series of breathing motions until a stable inflation is reached.

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* Aerospace Engineer, Space Applications and Technology Division.

† Senior Aerospace Engineer, Space Applications and Technology Division. Member AIAA.